2015 Spring - Information Theory

Homework 6 (Due to Apr.24)

Part 1: Tunstall Codes

Consider the following ternary DMS:$P_U(a) = 0.5$, $P_U(b) = 0.3$, $P_U(c) = 0.2$, and construct a binary block code($D = 2$) of length 4.

Part 2: The Variable-Length-to-Block Coding Theorm

Please prove (4.126) at page 120.

Part 3: Generation of discrete distribution from fair coins

Suppose we wish to generate a random variable X with distribution:$P_X(a) = 0.5$, $P_X(b) = 0.25$, $P_X(c) = 0.25$.
It's easy to guess the answer. If the first bit is 0, we let $X = a$, and 10$\rightarrow b$, 11$\rightarrow c$.
But there are many other possible same output distribution: 00$\rightarrow a$, 01$\rightarrow a$, 10$\rightarrow b$, 11$\rightarrow c$.

Consider a probability distribution on the leaves, such that the probability of a leaf at depth k is 2^{-k}. Please build the most efficient binary tree to generate a random variable X with distribution:$P_X(a) = 2/3$, $P_X(b) = 1/3$.

Part 4:Markov Chain

Consider a source with alphabet a, b, c. Aussume that
$P_{X2|X1}(a|a) = 0.7, P_{X2|X1}(b|a) = 0.2, P_{X2|X1}(c|a) = 0.1$
$P_{X2|X1}(a|b) = 0.1, P_{X2|X1}(b|b) = 0.8, P_{X2|X1}(c|b) = 0.1$
$P_{X2|X1}(a|c) = 0.1, P_{X2|X1}(b|c) = 0.3, P_{X2|X1}(c|c) = 0.6$

Please calculate X_4 and X_{inf} with initial state X_1 equal to [0.2,0.4,0.4].

Part 5: Matlab Exercise

Please construct the Tunstall's algorithm for optimum D-ary L-block encoding of a proper message set for an r-ary DMS.