2014 Fall - Introduction to Communiation System

Homework 1

Band limited signals are not also time limited

Provide a simple proof that, if

$$x(t)\prod(t/T) = x(t)$$

for some a and $X(f) = \mathfrak{F}[x(t)]$, then $\nexists\ F$ such that

$$X(f)\prod(f/F) = x(f)$$

.

What happens if we consider causal signals? that is

$$x(t)u_{-1}(t) = x(t)$$

.

Relationship between Fourier Series and Fourier Transform

The Fourier series is defined for periodic signals. The Fourier transform used for non-periodic signals. What is the relationship among the two?

Meaningfully define a Fourier series of a periodic signal with period that tends to infinity.

Similarly define the Fourier transforms of a periodic signal, that is of a power type signal.

In the first case construct a periodic signal from a non-periodic signal by replicating the values in the interval $[-T, +T]$ throughout $\mathbb{R}$. What is the Fourier series of such signal? What happens when $T \to \infty$?

It the second case construct an energy limited signal from a periodic signal $x(t)$ by multiplying it by a rectangular function $\prod(t/T)$ for some T integer multiple of the period T_0. What is the Fourier transform of such signal? What happens when $T \to \infty$?

Power content of a DSB-SC signal

In class we have seen that the power content of a DSB-SC signal can be approximated to

$$P_u = \frac{A_c^2}{2}P_m \tag{1}$$

where A_c is the carrier amplitude and P_m is the power of the modulated signal.

Verify the goodness of this approximation as a function of the signal with bandwidth W for the case where

- $m(t)$ is a rectangular function of width $1/W$
- $m(t)$ is a triangular function of width $1/W$
- $m(t)$ is a sinusoidal function of frequency W
- $m(t)$ is a sinc function of frequency W

Remember that you can use Parseval's relationship to determine this power.

Power content of a conventional AM signal

Determine the power content of a conventional AM signal as a function of the modulation index a, the carrier amplitude A_m and the power of the modulated signal P_m.

Evaluate the exact expression for the signals of the previous exercise.