Analog to digital conversion

Sampling and reconstruction

In the last lecture

- Non-Uniform quantizers
- Vector Quantizers
- Transformation of distributions

In the last lecture

- Why do we consider non uniform quantizers
- Why do we look at quantizers which take multiple bits in inputs
- Why do we want to map a distributions to the uniform distribution

In this lecture

- Mapping of quantized labels to a bit-stream
- Vector Quantizers
- Waveform coding

Encoding

How do we map the samples to the bits

Encoding

- As the number of quantization level increases, mapping samples to bits becomes more involved
  - Does in matter?
    - As far as we are transmitting over a noiseless channel, a one-to-one mapping is as good as another one
    - If we introduce noise, things change radically
      - We want the erroneous reconstruction to still be close to real values

Encoding

- A simple mapping is the one in which we simply consider the binary expansion of the quantization levels
  - Level 1 001
  - Level 2 010
  - Level 3 011
  - …

Encoding

- What can we do if we want to minimize the impact on the reconstruction error
  - In this case we want to minimize the difference between the bit representation of one label and the next one
  - This is not the case in the natural encoding
    - 7 -> 0 1 1 1
    - 8 -> 1 0 0 0

Encoding

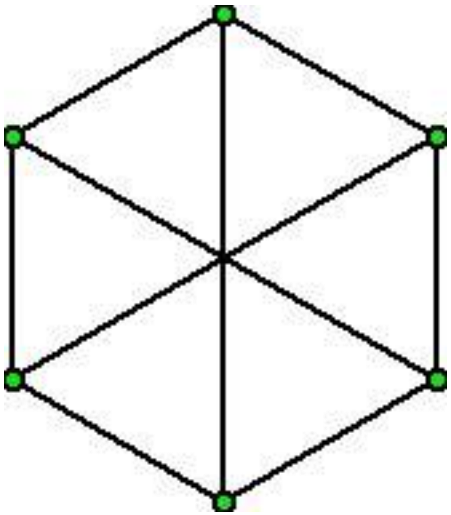
- Think about this problem:
  - This is a labelling, so we can settle for a table which maps the quantization label to a bit string

Encoding

- Think about this problem:
  - The minimum distance between two label is 1 bit
    - Can we attain this?
  - This at this problem in a geometrical manner

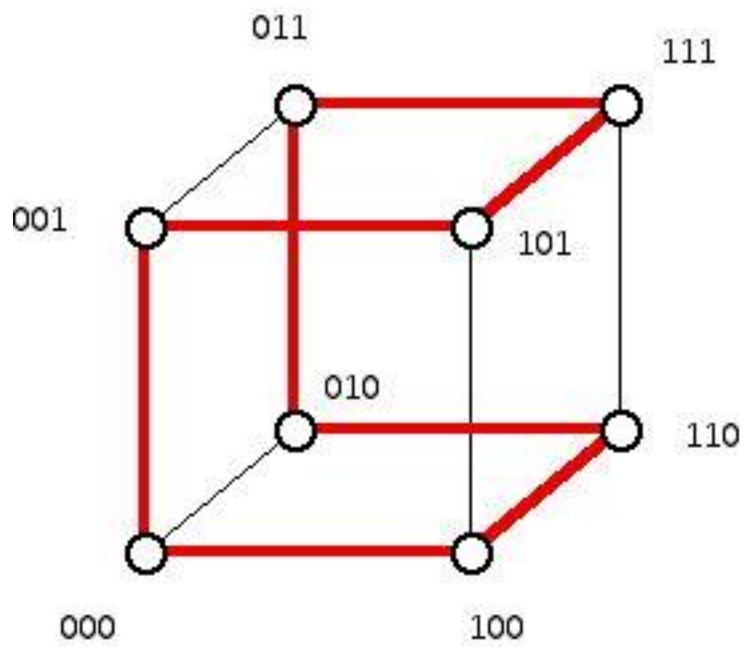
Encoding

- A code of length 3 in a cube on 3 dimensions



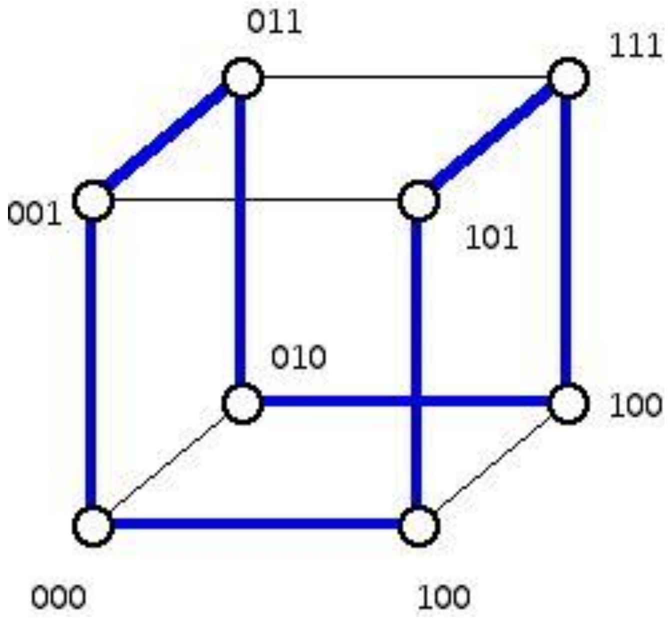
Encoding

- The code requirement on the label distance becomes a circuit on the vertexes on the cube



Encoding

- The code requirement on the label distance becomes a circuit on the vertexes on the cube



Encoding

TABLE 7.3 NBC AND GRAY CODES FOR A 16-LEVEL QUANTIZATION

Quantization level	Level order	NBC code	Gray code
$\hat{x}_1$	0	0000	0000
$\hat{x}_2$	1	0001	0010
$\hat{x}_3$	2	0010	0011
$\hat{x}_4$	3	0011	0001
$\hat{x}_5$	4	0100	0101
$\hat{x}_6$	5	0101	0100
$\hat{x}_7$	6	0110	0110
$\hat{x}_8$	7	0111	0111
$\hat{x}_9$	8	1000	1111
$\hat{x}_{10}$	9	1001	1110
$\hat{x}_{11}$	10	1010	1100
$\hat{x}_{12}$	11	1011	1101
$\hat{x}_{13}$	12	1100	1001
$\hat{x}_{14}$	13	1101	1000
$\hat{x}_{15}$	14	1110	1010
$\hat{x}_{16}$	15	1111	1011

Waveform Coding

How is voice coded in the phone line

Waveform Coding

- Having a statistical description of the input is not always possible
- Or sometimes the description is simply too complex or not too useful

Waveform Coding

- How do we design a system for these scenarios
  - We can simply focus on a wide class of signals that are characterized by some reasonable features

Waveform Coding

Pulse code modulation is the simplest and oldest waveform coding scheme. A pulse code modulator consists of three basic sections: a sampler, a quantizer and an encoder. A functional block diagram of a PCM system is shown in Figure 7.7. In PCM, we make the following assumptions:

1. The waveform (signal) is bandlimited with a maximum frequency of W. Therefore, it can be fully reconstructed from samples taken at a rate of $f_s = 2W$ or higher.
2. The signal is of finite amplitude. In other words, there exists a maximum amplitude $x_{\max}$ such that for all t, we have $|x(t)| \leq x_{\max}$.
3. The quantization is done with a large number of quantization levels N, which is a power of 2 ($N = 2^{\nu}$).

Waveform Coding

- System model

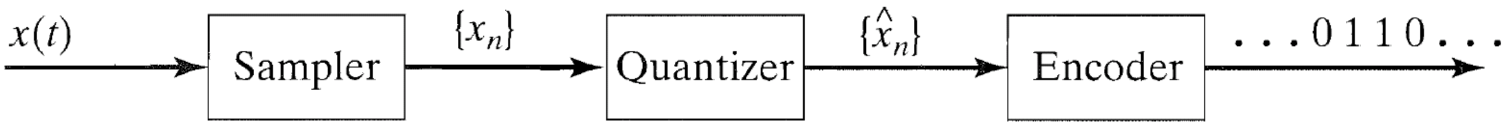


Figure 7.7 Block diagram of a PCM system.

Waveform Coding

- We can start focusing on uniform sources

Uniform PCM. In uniform PCM, we assume that the quantizer is a uniform quantizer. Since the range of the input samples is $[-x_{\max}, +x_{\max}]$ and the number of quantization levels is N, the length of each quantization region is given by

$$\Delta = \frac{2x_{\max}}{N} = \frac{x_{\max}}{2^{\nu-1}}. \tag{7.4.1}$$

Waveform Coding

- What is the distortion in this case?

The distortion introduced by quantization (quantization noise) is therefore

$$E[\tilde{X}^2] = \int_{-\frac{\Delta}{2}}^{+\frac{\Delta}{2}} \frac{1}{\Delta} \tilde{x}^2 d\tilde{x} = \frac{\Delta^2}{12} = \frac{x_{\max}^2}{3N^2} = \frac{x_{\max}^2}{3 \times 4^\nu}, \tag{7.4.2}$$

where ν is the number of bits/source sample and we have employed Equation (7.4.1). The signal-to-quantization noise ratio then becomes

$$\mathrm{SQNR} = \frac{P_X}{\overline{\tilde{X}^2}} = \frac{3 \times N^2 P_X}{x_{\max}^2} = \frac{3 \times 4^\nu P_X}{x_{\max}^2}, \tag{7.4.3}$$

Waveform Coding

- What is the relationship between the error and the quantization levels?

Expressing SQNR in decibels, we obtain

$$\text{SQNR}\Big|_{\text{dB}} \approx 10\log_{10}\frac{P_X}{x_{\max}^2} + 6\nu + 4.8. \tag{7.4.4}$$

We can see that each extra bit (increase in ν by one) increases the SQNR by 6 dB. This is a very useful strategy for estimating how many extra bits are required to achieve a desired SQNR.

Waveform Coding

Example 7.4.1

What is the resulting SQNR for a signal uniformly distributed on $[-1, 1]$, when uniform PCM with 256 levels is employed?

Solution We have $P_X = \int_{-1}^{1} \frac{1}{2}x^2 dx = \frac{1}{3}$. Therefore, using $x_{\max} = 1$ and $\nu = \log 256 = 8$, we have

$$\text{SQNR} = 3 \times 4^{\nu} \times P_X = 4^{\nu} = 65536 \approx 48.16 \text{ dB}. \quad ■$$

Waveform Coding

The usual method for performing nonuniform quantization[3] is to first pass the samples through a nonlinear element that compresses the large amplitudes (reduces the dynamic range of the signal) and then performs a uniform quantization on the output. At the receiving end, the inverse (expansion) of this nonlinear operation is applied to obtain the sampled value. This technique is called *companding* (*comp*ressing–exp*anding*). A block diagram of this system is shown in Figure 7.8.

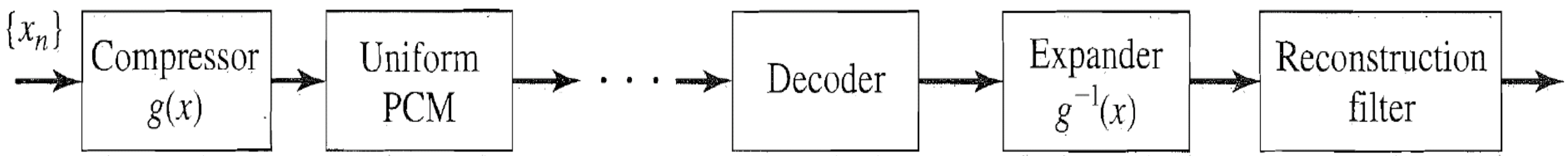


Figure 7.8 Block diagram of a nonuniform PCM system.

Waveform Coding

- In telephone systems we often use a pre/post processing of the information

There are two types of companders that are widely used for speech coding. The μ-law compander, used in the United States and Canada, employs the logarithmic function at the transmitting side, where $|x| \leq 1$:

$$g(x) = \frac{\log(1 + \mu|x|)}{\log(1 + \mu)} \operatorname{sgn}(x). \tag{7.4.7}$$

The parameter μ controls the amount of compression and expansion. The standard PCM system in the United States and Canada employs a compressor with $\mu = 255$ followed by a uniform quantizer with 8 bits/sample. Use of a compander in this system improves the performance of the system by about 24 dB. Figure 7.9 illustrates the μ-law compander characteristics for $\mu = 0, 5$, and 255.

Waveform Coding

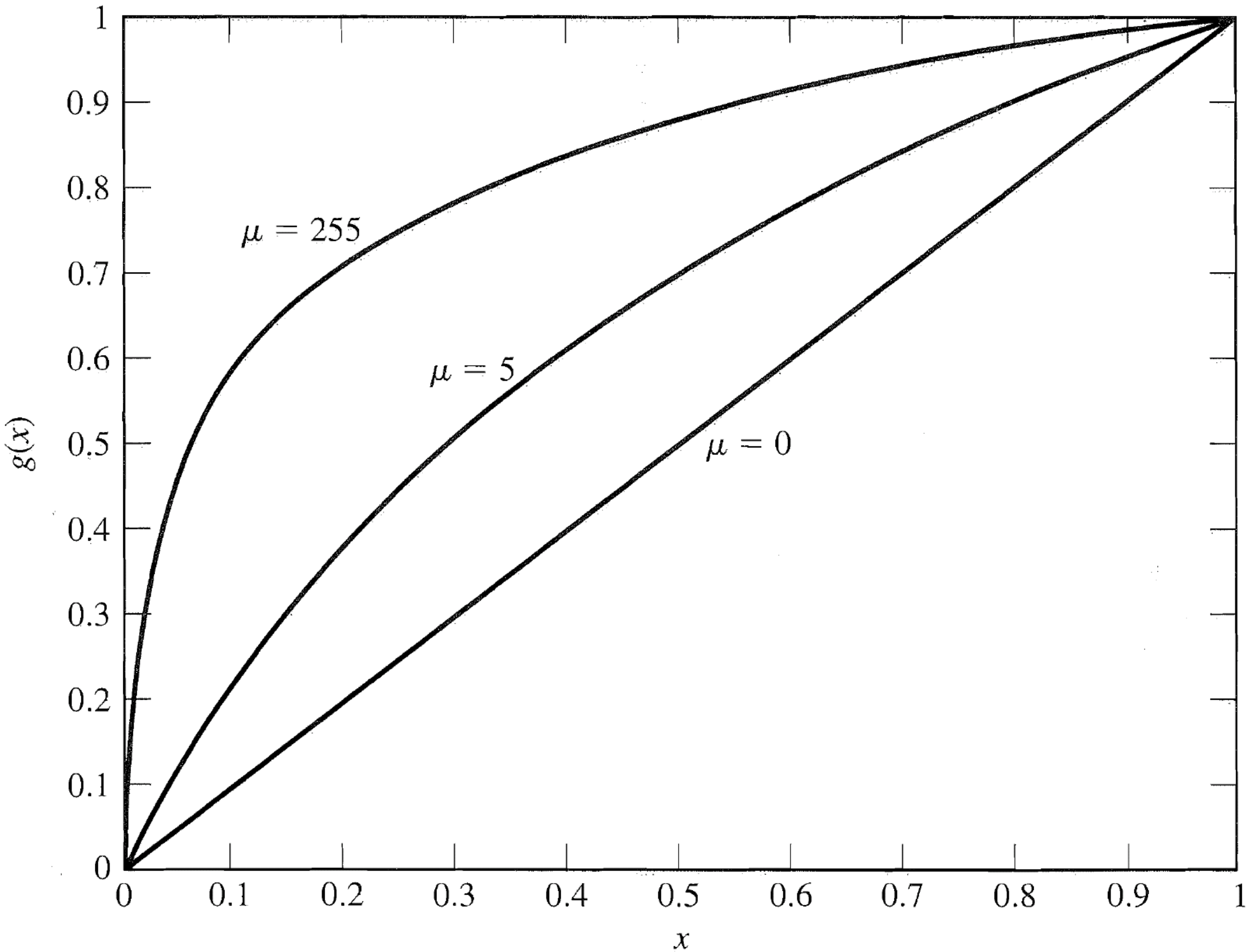


Figure 7.9 A graph of μ-law compander characteristics.

Waveform Coding

The second widely used logarithmic compressor is the A-law compander. The characteristics of this compander are given by

$$g(x) = \frac{1 + \log A|x|}{1 + \log A} \operatorname{sgn}(x), \tag{7.4.8}$$

where A is chosen to be 87.56. The performance of this compander is comparable to the performance of the μ-law compander. Figure 7.10 illustrates the characteristics of this compander for $A = 1, 2,$ and 87.56.

Waveform Coding

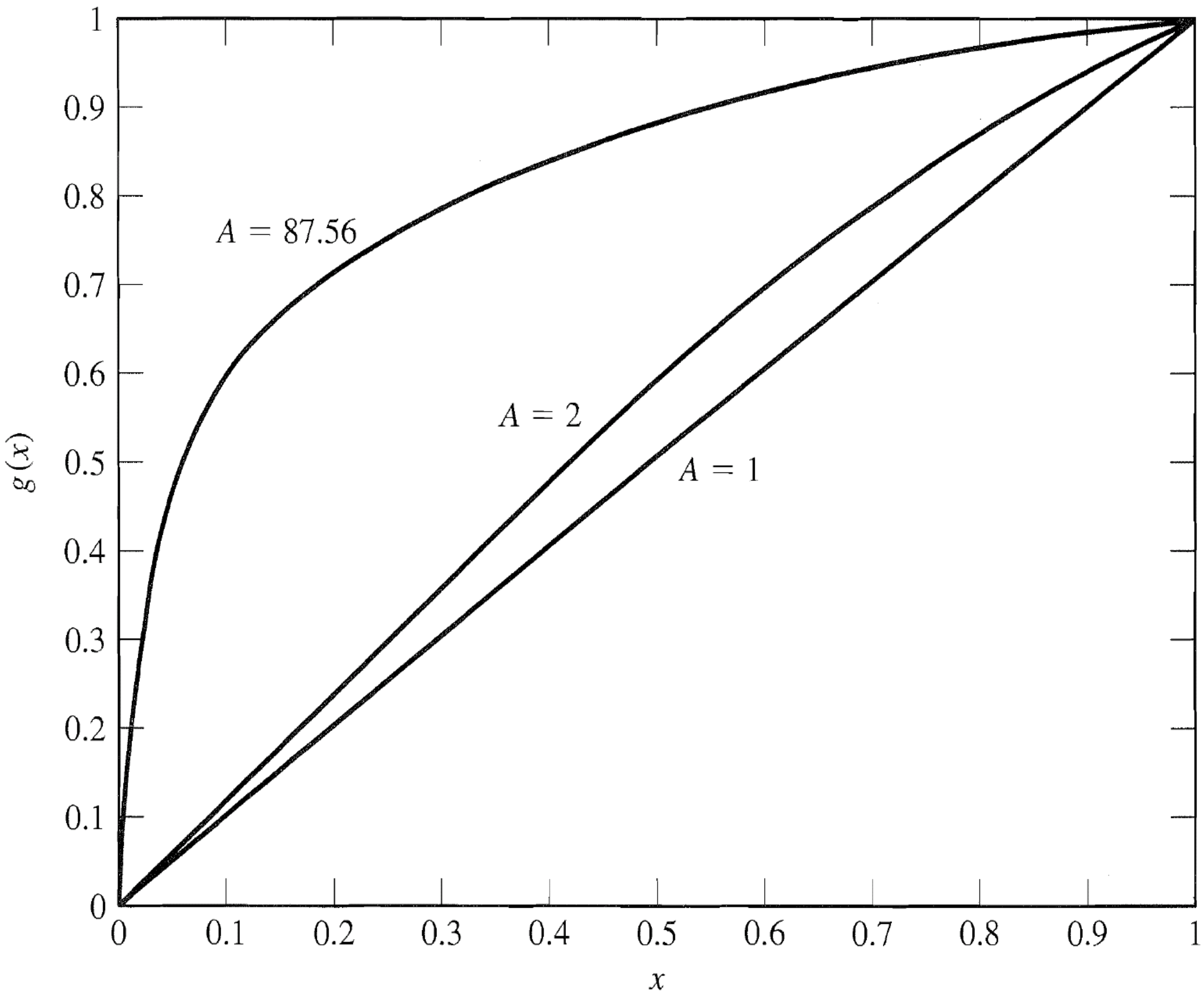


Figure 7.10 A graph of A-law compander characteristics.

That’s it for today!

I’ll see you next time